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Progress and challenges in the development of the composite wedge localization element

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Abstract

The composite wedge localization element:

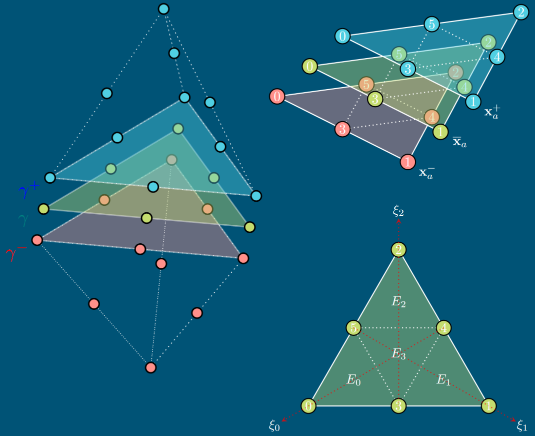
- Composite tetrahedron compatible.
- Regularizes sub-grid localization.
- Extending to fracture and failure.

Progress:

- Lower-order projections.
- Rigid-body modes.

Challenges:

- Pressure field stability.
- Decoupling length scales.
- Implicit solve convergence.



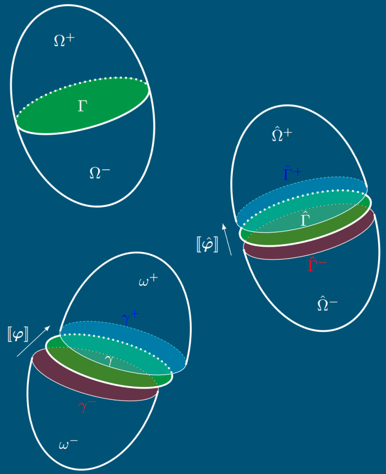
Background

Energy functional

With both solid and localization elements [1–3]:

$$
\begin{aligned}
\Pi[\varphi, \overline{\mathbf{F}}, \overline{\mathbf{P}}] = & \sum_{\pm} \int_{\Omega^{\pm}} A(\overline{\mathbf{F}}, \mathbf{Z}) \, dV + \int_{\Gamma} A(\overline{\mathbf{F}}, \mathbf{Z}) h \, dS \\
& + \sum_{\pm} \int_{\Omega^{\pm}} \overline{\mathbf{P}} : (\mathbf{F} - \overline{\mathbf{F}}) \, dV + \int_{\Gamma} \overline{\mathbf{P}} : (\mathbf{F} - \overline{\mathbf{F}}) h \, dS \\
& - \sum_{\pm} \int_{\Omega^{\pm}} \rho_0 \mathbf{B} \cdot \varphi \, dV - \sum_{\pm} \int_{\partial_{\mathbf{T}} \Omega^{\pm}} \mathbf{T} \cdot \varphi \, dS
\end{aligned}
$$

- Lagrange multiplier $\overline{\mathbf{P}}$ enforces $\overline{\mathbf{F}} = \mathbf{F}$, where $\overline{\mathbf{P}} = \mathbf{P} = \partial A / \partial \mathbf{F}$ is also enforced.
- Localization element thickness h required for both integration and normalization.



Localization kinematics

Let $\mathbf{x}(t) = \varphi(\boldsymbol{\xi}; t)$ and $\mathbf{X} = \mathbf{x}(0) = \varphi_0(\boldsymbol{\xi})$.

- Deformation from jump [4]:

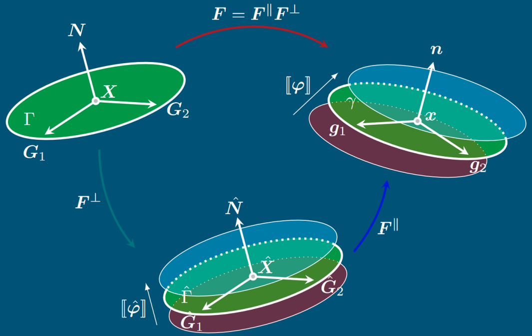
$$\mathbf{F}^{\perp} = \mathbf{I} + \frac{[\![\hat{\varphi}]\!]}{h} \otimes \mathbf{N} \quad \text{and} \quad [\![\varphi]\!] = \mathbf{F}^{\parallel}[\![\hat{\varphi}]\!]$$

- Deformation from surface:

$$\mathbf{F}^{\parallel} = \partial_\mu \varphi \otimes \partial^\mu \varphi_0 + \mathbf{n} \otimes \mathbf{N}$$

- Resulting additive decomposition:

$$\mathbf{F} = \mathbf{F}^{\parallel}\mathbf{F}^{\perp} = \mathbf{F}^{\parallel} + \frac{[\![\varphi]\!]}{h} \otimes \mathbf{N}$$



Fundamentally different from cohesive surface elements [5].

Element discretization

Let $\tilde{\mathbf{x}}_a = \frac{1}{2}(\mathbf{x}_a^+ + \mathbf{x}_a^-)$ and $[\![\mathbf{x}_a]\!] = \mathbf{x}_a^+ - \mathbf{x}_a^-$.

- Subtriangles project to linear element:

$$\bar{\mathbf{A}} = \lambda_\alpha \left(\int_{\Gamma_E} \lambda_\alpha \lambda_\beta \mathbf{I} \, dS \right)^{-1} \int_{\Gamma_E} \lambda_\beta \mathbf{A} \, dS$$

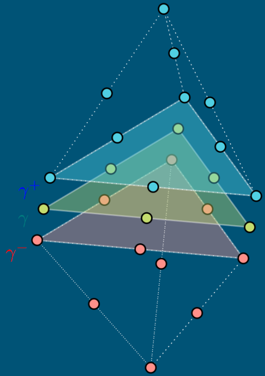
- Projected gradient operators:

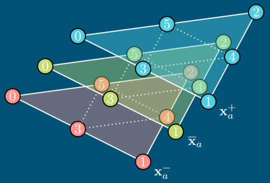
$$\bar{\mathbf{F}} = \bar{\boldsymbol{\mathcal{B}}}_a^{\|} \tilde{\mathbf{x}}_a + \bar{\boldsymbol{\mathcal{B}}}_a^{\perp} [\![\mathbf{x}_a]\!]$$

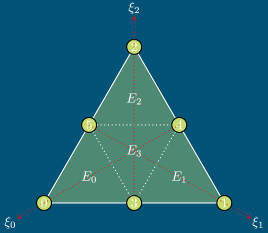
- Nodal forces, quasi-traction-separation:

$$f_a^{\pm} = \frac{1}{2} \int_\Gamma \mathbf{P} : \bar{\boldsymbol{\mathcal{B}}}_a^{\|} \, h \, dS \pm \int_\Gamma \mathbf{P} \bar{\mathbf{N}}_a \, dS$$

Implemented in Sierra/SolidMechanics [6].







Progress

Lower-order projections

Volumetric locking:

- Observed in nearly incompressible flow.
- Manifested as oscillatory pressure fields.

Mitigation technique [3]:

- Lower-order projection of the Jacobian.

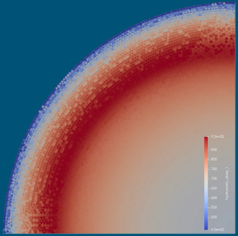
$$\tilde{\mathbf{F}} = \left(\frac{\bar{J}^{\star}}{\bar{J}} \right) \bar{\mathbf{F}} \quad \text{and} \quad \bar{J}^{\star} = \frac{1}{V_{\Omega}} \int_{\Omega} \bar{J} \, dV$$

- Lower-order projection of the pressure.

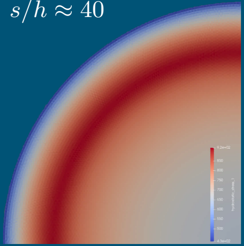
$$\bar{p}^{\star} = \frac{1}{\bar{J}^{\star} V_{\Omega}} \int_{\Omega} \frac{\operatorname{tr}(\tilde{\mathbf{P}} \tilde{\mathbf{F}}^{T})}{3} \, dV$$

- Corresponding adjusted nodal forces.

$$\mathbf{f}_a = \int_{\Omega} \left(\tilde{\mathbf{P}} - \frac{1}{3} \operatorname{tr}(\tilde{\mathbf{P}} \tilde{\mathbf{F}}^{T}) \tilde{\mathbf{F}}^{-T} + \bar{J} \bar{p}^{\star} \tilde{\mathbf{F}}^{-T} \right) \cdot \bar{\boldsymbol{\mathcal{B}}}_a \left(\frac{\bar{J}^{\star}}{\bar{J}} \right)^{1/3} dV$$



$s/h \approx 40$



Rigid-body modes

Surface-separating finite elements:

- Localization elements.
- Cohesive surface elements.

Composite surface-separating finite elements:

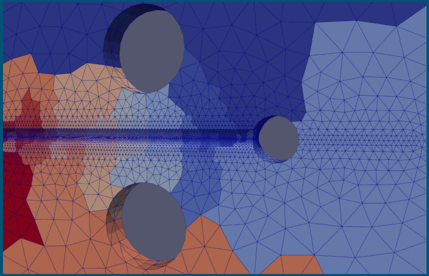
- Additional rigid-body modes.

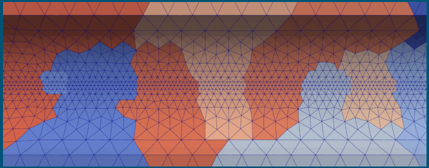
Using lower-order volumetric projections:

- Additional low-energy modes?
- Stabilization is turned off here.

Parallel decompositions:

- No rigid-body modes in the final assembly as long as the ham stays in the sandwich.
- Need info across processor boundaries.





Challenges

Pressure field stability

Large ratios of s/h disrupts pressure fields:

- Oscillatory or downright nasty.
- Visible effects after significant plasticity.
- Refinement typically alleviates the issue.

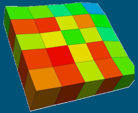
An issue for any localization element, so far:

- Hexahedral localization element.
- Composite wedge localization element.

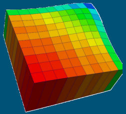
Is there always a point of instability?

$s/h \approx 4$

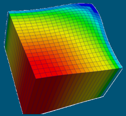
$s/h = 24$



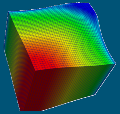
$s/h = 12$

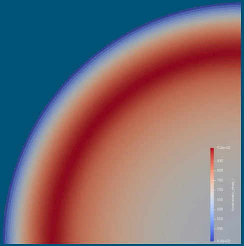


$s/h = 6$

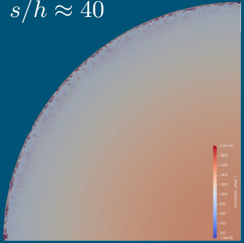


$s/h = 3$





$s/h \approx 40$



Decoupling length scales

Weight contributions separately [7]:

$$f_a^{\pm} = \frac{1}{2}\int_\Gamma \mathbf{P} : \bar{\boldsymbol{\mathcal{B}}}_a^{\parallel}\, t\, dS \pm \int_\Gamma \mathbf{P}\bar{\mathbf{N}}_a\, dS$$

Or try to explicitly retain variational structure:

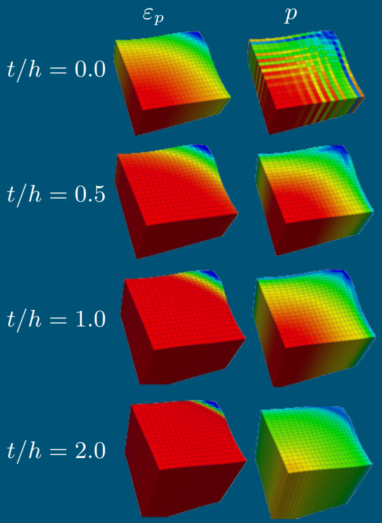
$$\int_\Gamma A(\mathbf{F}^{\parallel}, \mathbf{Z})\, t\, dS + \int_\Gamma \left[A(\mathbf{F}, \mathbf{Z}) - A(\mathbf{F}^{\parallel}, \mathbf{Z})\right] h\, dS$$

Surface element, quasi-traction-separation, extra:

$$f_a^{\pm} = \frac{1}{2}\int_\Gamma \mathbf{P}^{\parallel} : \bar{\boldsymbol{\mathcal{B}}}_a^{\parallel}\, t\, dS \pm \int_\Gamma \mathbf{P}\bar{\mathbf{N}}_a\, dS$$

(ignore?) $\pm \int_\Gamma \left(\mathbf{P} - \mathbf{P}^{\parallel}\right) : \bar{\boldsymbol{\mathcal{B}}}_a^{\parallel}\, h\, dS$

Is any of this fair in the first place?



Implicit solve convergence

Explicit integration analyses:

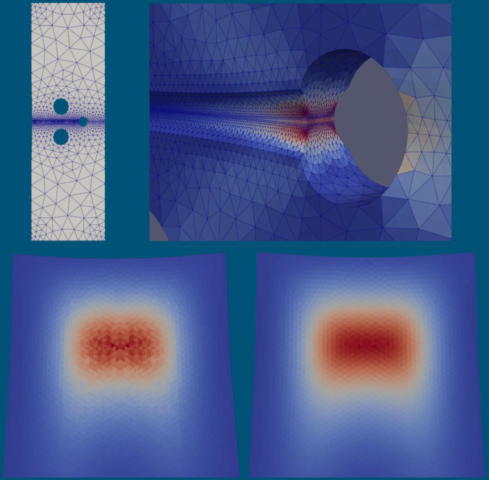
- Complicated by massless elements.
- Less desirable in certain cases.

Implicit integration analyses:

- Sometimes the fields look great,
  and the damage evolution is "smooth"
  but it just will not converge!
- Currently a work-in-progress [7–10].

Failure modeling is hard! Who knew?

- Need more refinement?
- Need non-local damage model?
- Something else happening?



Conclusion

The composite wedge localization element:

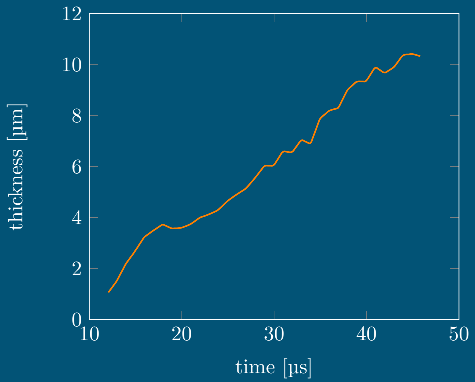
- Composite tetrahedron compatible.
- Original development finished previously.
- Newly implemented lower-order projections.

Ratio of element size to thickness (s/h) issue:

- Is mesh refinement always possible?
- Will scaling the membrane forces work?
  - Which way should they be scaled?
- Does h need to grow as a field [4]?

Implicit integration analyses:

- Is there something preventing convergence?
- Or is this simply a difficult problem to solve?



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