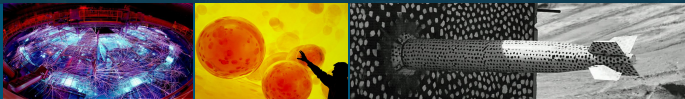
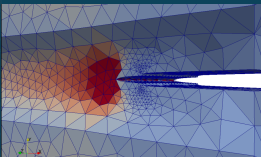




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Developing a composite wedge localization element

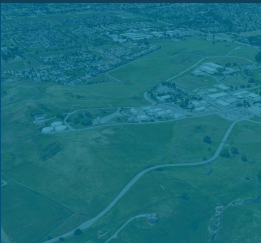


17th U.S. National Congress on Computational Mechanics

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In computational solid mechanics...

- It is non-trivial to attain a reasonable mesh and solution.
 - Tetrahedral elements are easy to mesh, but can perform poorly.
 - Hexahedral elements can perform better, but are hard to mesh.
 - Composite tetrahedral elements offer the best of both worlds [1, 2, 3].
- Highly localized deformation affects meshing and solving.
 - Increased mesh refinement does not converge upon a solution.
 - Cohesive zone elements provide convergence using a special constitutive law [4].
 - Localization elements provide convergence using the material constitutive law [5].

A composite wedge localization element is developed for fracture and failure [6].



Abstract

Finite element formulation

- Energy functional

- Localization kinematics

- Element discretization

- Element projection

Finite element applications

- Calibration to fracture toughness

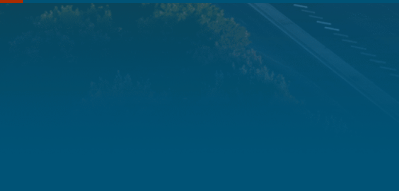
- Three-hole tension experiment

Conclusion

References

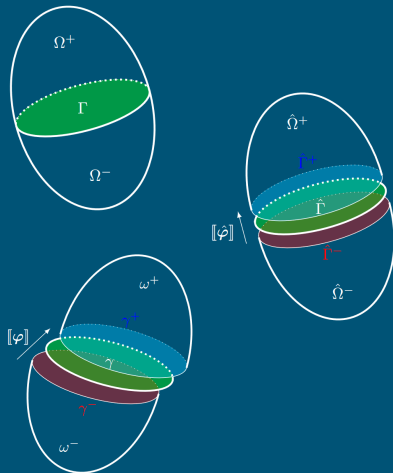


Finite element formulation



5 Energy functional

$$\begin{aligned}\Pi_0[\boldsymbol{\varphi}] = & \underbrace{\int_{\Omega} A(\mathbf{F}, \mathbf{Z}) dV}_{\text{orange}} - \int_{\Omega} \rho_0 \mathbf{B} \cdot \boldsymbol{\varphi} dV - \int_{\partial_{\mathbf{T}} \Omega} \mathbf{T} \cdot \boldsymbol{\varphi} dS \\ & \underbrace{\int_{\Gamma} \phi(\mathbf{F}, \mathbf{Z}) dS}_{\text{blue}} + \sum_{\pm} \int_{\Omega^{\pm}} A(\mathbf{F}, \mathbf{Z}) dV \\ & \underbrace{\int_{\Gamma} A(\mathbf{F}, \mathbf{Z}) h dS}_{\text{green}} \\ & \int_{\Gamma} A^{\parallel}(\mathbf{F}, \mathbf{Z}) h^{\parallel} dS + \int_{\Gamma} A^{\perp}([\![\boldsymbol{\varphi}]\!], \mathbf{Z}) h^{\perp} dS\end{aligned}$$



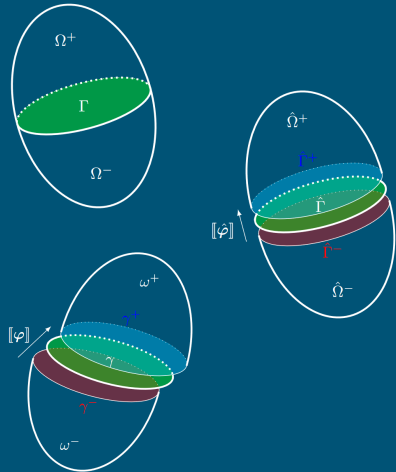
Length scale h normalizes $[\![\boldsymbol{\varphi}]\!]$, the displacement jump [5].

Using two length scales $h^{\parallel}$, $h^{\perp}$ will also decouple the membrane forces [6].

6 Energy functional

$$\begin{aligned}\Pi[\boldsymbol{\varphi}, \bar{\mathbf{F}}, \bar{\mathbf{P}}] &= \sum_{\pm} \int_{\Omega^{\pm}} A(\bar{\mathbf{F}}, \mathbf{Z}) dV + \int_{\Gamma} A(\bar{\mathbf{F}}, \mathbf{Z}) h dS \\ &+ \sum_{\pm} \int_{\Omega^{\pm}} \bar{\mathbf{P}} : (\mathbf{F} - \bar{\mathbf{F}}) dV + \int_{\Gamma} \bar{\mathbf{P}} : (\mathbf{F} - \bar{\mathbf{F}}) h dS \\ &- \sum_{\pm} \int_{\Omega^{\pm}} \rho_0 \mathbf{B} \cdot \boldsymbol{\varphi} dV - \sum_{\pm} \int_{\partial_{\mathbf{T}} \Omega^{\pm}} \mathbf{T} \cdot \boldsymbol{\varphi} dS\end{aligned}$$

Equivalent functional via Lagrange multiplier $\bar{\mathbf{P}}$ enforcing the constraint $\bar{\mathbf{F}} = \mathbf{F}$ where, $\bar{\mathbf{P}} = \mathbf{P} = \partial A / \partial \mathbf{F}$ is also enforced [1, 2, 3].



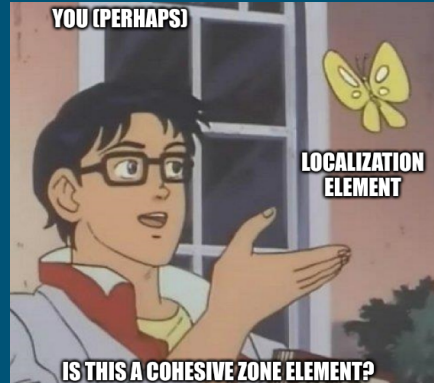
Permits formulation of composite elements.

Formulate $\mathbf{F}$ for localization element to calculate $\mathbf{P}$ using the constitutive model for A .

7 Localization kinematics



- Element construction and kinematics are similar between the two elements.
- The constitutive behaviors are not.
 - Cohesive zone elements prescribe a traction separation law [4].
 - Localization elements use the same constitutive law as the bulk [5].
 - They accomplish this by introducing a length scale (or two).



Important

Localization elements are not cohesive zone elements.

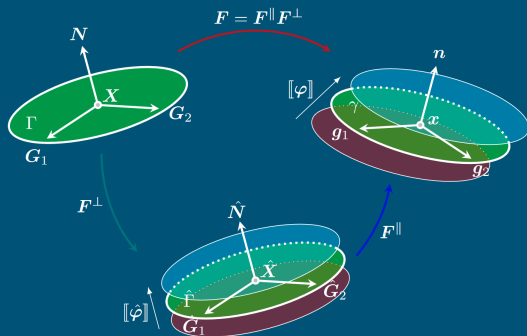
Let $\mathbf{x}(t) = \varphi(\boldsymbol{\xi}; t)$ and $\mathbf{X} = \mathbf{x}(0) = \varphi_0(\boldsymbol{\xi})$.

$$\mathbf{F}^\perp = \mathbf{I} + \frac{[\hat{\boldsymbol{\varphi}}]}{h} \otimes \mathbf{N}, \quad [\boldsymbol{\varphi}] = \mathbf{F}^\parallel [\hat{\boldsymbol{\varphi}}]$$

$$\mathbf{F}^\parallel = \underbrace{\mathbf{g}_\mu}_{\partial_\mu \boldsymbol{\varphi}} \otimes \underbrace{\mathbf{G}^\mu}_{\partial^\mu \boldsymbol{\varphi}_0} + \mathbf{n} \otimes \mathbf{N}$$

$$\mathbf{n} = \frac{\mathbf{g}_1 \times \mathbf{g}_2}{\|\mathbf{g}_1 \times \mathbf{g}_2\|}, \quad \mathbf{N} = \frac{\mathbf{G}_1 \times \mathbf{G}_2}{\|\mathbf{G}_1 \times \mathbf{G}_2\|}$$

$$\mathbf{F} = \mathbf{F}^\parallel + \frac{[\boldsymbol{\varphi}]}{h} \otimes \mathbf{N}$$



Additive decomposition

Frame invariant deformation gradient, order of multiplicative decomposition irrelevant [5, 6].

9 Element discretization

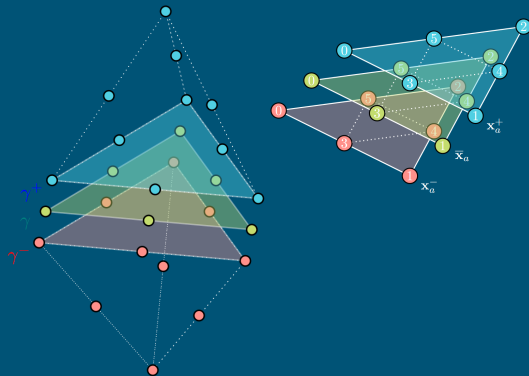


Let $\tilde{\mathbf{x}}_a = \frac{1}{2}(\mathbf{x}_a^+ + \mathbf{x}_a^-)$ and $[[\mathbf{x}_a]] = \mathbf{x}_a^+ - \mathbf{x}_a^-$.

$$\varphi(\boldsymbol{\xi}) = N_a(\boldsymbol{\xi})\tilde{\mathbf{x}}_a, \quad [[\varphi]] = N_a(\boldsymbol{\xi})[[\mathbf{x}_a]]$$

$$\mathbf{F} = \underbrace{\mathcal{B}_a^{\parallel}}_{f(\tilde{\mathbf{x}}_a)} \tilde{\mathbf{x}}_a + \mathcal{B}_a^{\perp} [[\mathbf{x}_a]] = \sum_{\pm} \mathcal{B}_a^{\pm} \mathbf{x}_a^{\pm}$$

$$\mathbf{R}_a^{\pm} = \underbrace{\frac{h^{\parallel}}{2}}_{h^{\parallel} \neq h} \int_{\Gamma} \bar{\mathbf{P}} : \mathcal{B}_a^{\parallel} dS \pm \underbrace{\frac{h^{\perp}}{h}}_{h^{\perp} = h} \int_{\Gamma} \bar{\mathbf{P}} \mathbf{N} N_a dS$$



Nodal forces

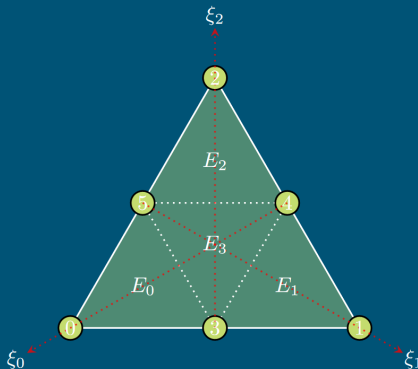
Independent control of localization length scale [6], inherently defined traction-separation [5].

Projection of subtriangle fields onto linear element

$$\bar{\mathbf{A}} = \lambda_\alpha \left(\int_{\Gamma_E} \lambda_\alpha \lambda_\beta \mathbf{I} dS \right)^{-1} \int_{\Gamma_E} \lambda_\beta \mathbf{A} dS$$

$$\mathbf{R}_a^\pm = \frac{h^\parallel}{2} \int_\Gamma \mathbf{P} : \bar{\mathbf{B}}_a^\parallel dS \pm \frac{h^\perp}{h} \int_\Gamma \mathbf{P} \bar{\mathbf{b}}_a^\perp dS$$

$$\bar{\mathbf{B}}_{a;ijk}^\pm = \frac{1}{2} \left(\delta_{ik} \bar{b}_{a;j}^{\parallel,0} + \varepsilon_{ijk} \underbrace{\bar{b}_{a;j}^{\parallel,j}}_{f(\tilde{\mathbf{x}}_a)} \right) \pm \delta_{ik} \bar{b}_{a;j}^\perp$$

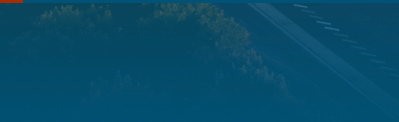


Consistent with 10-noded composite tetrahedron

Nearly incompressible large deformation plasticity [2, 3], localization and possible fracture [6].

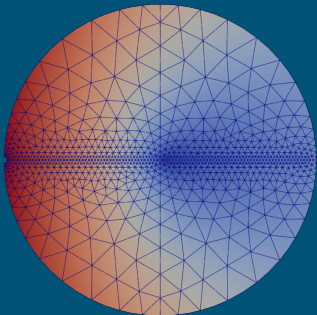


Finite element applications



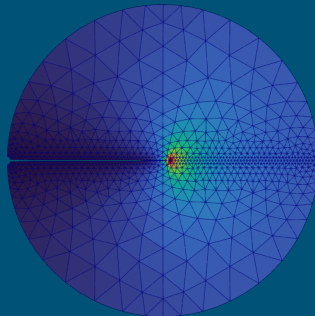
Hill plasticity [7] damage model [8] for Al 6061-T6 calibrated to smooth/notched tension [9].

Apply K_I displacement field.



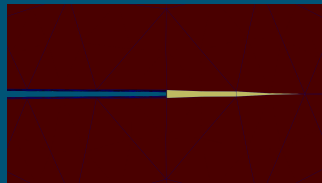
Displacement (magnitude).

Ramp K_I up until K_{IC} .



True stress (magnitude).

Calibrate h to fracture.



Damage, onset of fracture.

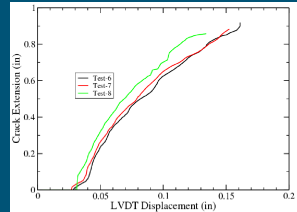
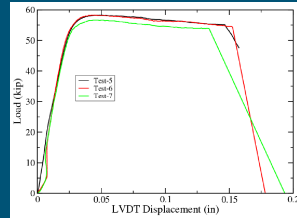
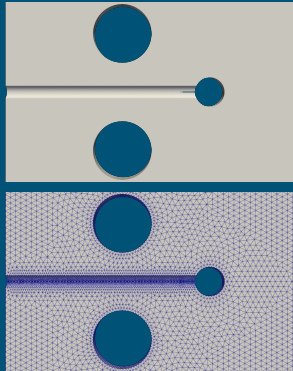
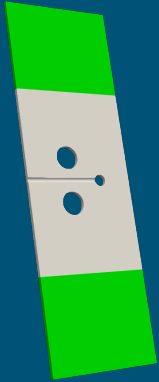
Current questions:

- Scale of h [5]?
- Element locking [3]?
- Stiff problem?

13 Three-hole tension experiment



Stable ductile fracture along a known path [10].





A new 12-noded composite wedge localization element.

- Compatible with the 10-noded composite tetrahedral element.
- Regularizes localized deformation using one (or two) length scales.

Applicable to fracture and failure in finite element models.

- Calibration of length scale to fracture toughness.
- Prediction of stable ductile fracture along a predetermined path.

Ductile fracture along unknown paths presents several challenges.

- Criterion for crack advancement with complicated material models.
- Remeshing and placement of localization elements (FRANC3D).



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